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On spontaneous CP-violation in the Higgs sector

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Abstract. We consider the behaviour of the axion mass as a function of Higgs coupling constants. The analysis is significantly simplified when we identify the axion field with the phase difference of the Higgs neutral components. Spontaneous CP-violation is induced by VEV of the axion field. The estimation of the axion mass for any values of $\delta_{\rm CP}$ and $\tan\beta$ shows inconsistency of MSSM with the present experimental results.

1 Introduction

One of the more elegant ways of introducing CP-violation is based on the possibility of spontaneous T-breaking in the system of two interacting Higgs doublets, developed by T.D. Lee [1]. This system naturally arises in the minimal supersymmetric model and has been shown to render a CP-violating vacuum even though the Lagrangian itself is CP-conserving. It becomes possible when one-loop corrections to the Higgs potential are taken into account [2],[3], leading to a term in the VEV of the potential which depends on the relative phase of the Higgs fields. The viability of this theory can be determined through its phenomenological implications, one of which is the neutral Higgs mass spectrum. As shown below, we can directly relate the CP-violating angle to the vacuum expectation value of the axion. This fact provides us with an opportunity to explicitly examine the dynamics of the axion mass and to simplify the analysis of the physical spectrum.

The most general renormalizable CP-conserving potential of two Higgs doublets is given by

$$\begin{aligned} V(\Phi_1,\Phi_2) = {} & m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 - (m_3^2\Phi_1^\dagger\Phi_2 + \text{h.c.}) \\ & +\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ & +\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{1}{2}[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}] \\ & +\frac{1}{2}\{\Phi_1^\dagger\Phi_2[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)] + \text{h.c.}\}\,, \quad (1) \end{aligned}$$

where all parameters are real. After spontaneous symmetry breaking, the neutral components of the Higgs fields acquire the following vacuum expectation values

$$\langle\Phi_1^0\rangle = v_1, \quad \langle\Phi_2^0\rangle = v_2 \mathrm{e}^{\mathrm{i}\delta}\,.$$

It has been shown [2] that the VEV of the potential for non-zero λ_5 can be rewritten as follows

$$\begin{aligned} \langle V\rangle = {} & m_1^2v_1^2 + m_2^2v_2^2 + 2\lambda_5v_1^2v_2^2 \\ & \times\left\{(\cos\delta-\Delta)^2 - \Delta^2\right\} + \lambda_1v_1^4 \\ & +\lambda_2v_2^4 + (\lambda_3+\lambda_4-\lambda_5)v_1^2v_2^2\,, \end{aligned} \tag{2}$$

where

$$\Delta = \frac{2m_3^2 - \lambda_6v_1^2 - \lambda_7v_2^2}{4\lambda_5v_1v_2}\,. \tag{3}$$

(We deviate from the notation of [2] in the definitions of the $v_{1,2}$ and $\lambda_{5\text{–}7}$.) From (2) one can see that in order for the spontaneous CP-breaking to occur the following inequalities must hold

$$\lambda_5 > 0\,, \tag{4}$$

$$-1 < \Delta < 1\,. \tag{5}$$

In this case we obtain a non-zero CP-violating phase δ given by

$$\cos\delta_{\rm CP} = \frac{2m_3^2 - \lambda_6v_1^2 - \lambda_7v_2^2}{4\lambda_5v_1v_2}\,. \tag{6}$$

The requirement that our vacuum be at least a stationary point of the potential results in the following constraints

$$m_1^2 = -2\lambda_1v_1^2 - (\lambda_3+\lambda_4-\lambda_5)v_2^2 - \lambda_6v_1v_2\cos\delta\,, \tag{7}$$

$$m_2^2 = -2\lambda_2v_2^2 - (\lambda_3+\lambda_4-\lambda_5)v_1^2 - \lambda_7v_1v_2\cos\delta\,, \tag{8}$$

$$0 = \sin\delta\left(m_3^2 - 2\lambda_5v_1v_2\cos\delta - \frac{1}{2}\lambda_6v_1^2 - \frac{1}{2}\lambda_7v_2^2\right), \tag{9}$$

where the last condition coincides with (6) in the case of the CP-violating vacuum.

2 The axion mass

To calculate the mass spectrum of the system we should choose the appropriate coordinates. The vacuum can be described by the three variables $v_{1,2}$ and δ, so it is natural to go over to the polar coordinates:

$$\Phi_k(x) = \rho_k(x)\mathrm{e}^{\mathrm{i}\xi_k(x)}\,, \quad k = 1,2\,. \tag{10}$$

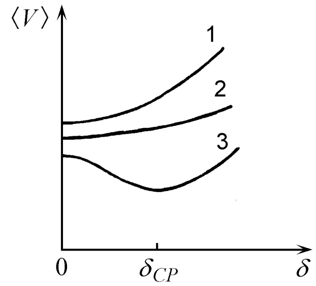

Fig. 1. This graph illustrates the qualitative behaviour of $\langle V\rangle$ as a function of δ (for simplicity we set $\lambda_{6,7}=0$): 1 – $\lambda_5 < \lambda_{\rm crit} = m_3^2/2v_1v_2$, 2 – $\lambda_5 = \lambda_{\rm crit}$, 3 – $\lambda_5 > \lambda_{\rm crit}$

Further, let us consider the behaviour of the fields near the vacuum

$$\rho_k = v_k + \eta_k(x)$$
$$\xi_k(x) = \bar\delta_k + \delta_k(x)$$

We would like to associate a particle with every real degree of freedom, therefore we should check whether the kinetic term has the proper form. In terms of the new variables the kinetic term reads

$$\begin{aligned}\partial_\mu \Phi_k^\dagger \partial^\mu \Phi_k &= \partial_\mu\eta_k\partial^\mu\eta_k + v_1^2\partial_\mu\delta_1\partial^\mu\delta_1 + v_2^2\partial_\mu\delta_2\partial^\mu\delta_2 \\ &\quad +(\text{higher order terms}) \\ &= \partial_\mu\eta_k\partial^\mu\eta_k + \partial_\mu\alpha\partial^\mu\alpha + \partial_\mu\gamma\partial^\mu\gamma \\ &\quad +(\text{higher order terms}) \end{aligned} \tag{11}$$

where we have introduced

$$\begin{aligned}\alpha &= \frac{v_1v_2}{v}\left(\delta_2-\delta_1\right), \\ \gamma &= \frac{v_1^2}{v}\delta_1 + \frac{v_2^2}{v}\delta_2, \\ v^2 &= v_1^2+v_2^2 .\end{aligned} \tag{12}$$

The potential depends upon a difference of the Higgs' phases solely, so we can interpret γ as a Goldstone boson appearing due to spontaneous U(1) breaking. We may assume that the real parts of $\Phi^0_{1,2}$ are CP-even fields [4] then the remaining angular degree of freedom, proportional to δ, corresponds to a CP-odd axion, while the linear combinations of ρ_1 and ρ_2 give two CP-even (in the case of CP-conserving vacuum) Higgs bosons h_0 and H_0. The mass spectrum can be found by diagonalization of the matrix of the second derivatives of the potential:

$$\begin{aligned} M_{ij}^2 &= \frac{1}{2}\left.\frac{\partial^2\langle V\rangle}{\partial a_i\partial a_j}\right|_{\rm vac}, \\ a_i &= (v_1, v_2, \alpha) .\end{aligned} \tag{13}$$

In the case of relatively small λ_5, i.e. when condition (5) cannot be fulfilled, the minimum of the potential is given by $\delta_{\rm CP}=0$ and CP-even and CP-odd state do not mix:

$$\left.\frac{\partial^2\langle V\rangle}{\partial v_{1,2}\partial\alpha}\right|_{\alpha=0} = 0$$

and when $\lambda_{5–7}=0$ we recover the well-known result

$$M_{A_0}^2 = \frac{1}{2}\left.\frac{\partial^2\langle V\rangle}{\partial\alpha^2}\right|_{\alpha=0} = \frac{2m_3^2}{\sin 2\beta}, \tag{14}$$

where $\tan\beta = v_2/v_1$. In the limit $m_3 \to 0$ the PQ-symmetry of the potential is restored, resulting in the massless axion. The other two mass-eigenvalues correspond to h_0 and H_0 and are given, for example, in [5]. In general, the axion remains massive until the parameters $\lambda_{5,6,7}$ reach certain critical values, determined by $4\lambda_5v_1v_2 = 2m_3^2 - \lambda_6v_1^2 - \lambda_7v_2^2$. The curvature of the potential vanishes:

$$\begin{aligned} M_{A_0}^2 &= \frac{1}{2}\left.\frac{\partial^2\langle V\rangle}{\partial\alpha^2}\right|_{\alpha=0} \propto 2m_3^2 - 4\lambda_5v_1v_2 - \lambda_6v_1^2 - \lambda_7v_2^2 \\ &= 0 \end{aligned} \tag{15}$$

giving rise to a massless axion. The further increase of λ_5 makes the curvature matrix flip sign, signifying $\delta_{\rm CP}=0$ is not a stable stationary point any more and spontaneous CP-violation occurs. To get a physical boson mass spectrum we must expand all the fields around the CP-breaking vacuum. Now the axion field acquires VEV, as dictated by (6), even though it is not a mass-eigenstate any more and, generally speaking, all matrix elements M_{ij}^2 are non-zero:

$$\begin{aligned} m_{ij}^2 &\equiv \frac{1}{v^2}M_{ij}^2, \\ m_{11}^2 &= 4\lambda_1\cos^2\beta + 2\lambda_5\sin^2\beta\cos^2\delta + \lambda_6\sin 2\beta\cos\delta, \\ m_{12}^2 &= \cos\delta(\lambda_6\cos^2\beta + \lambda_7\sin^2\beta) \\ &\quad + \sin 2\beta(\lambda_3+\lambda_4-\lambda_5\sin^2\delta), \\ m_{22}^2 &= 4\lambda_2\sin^2\beta + 2\lambda_5\cos^2\beta\cos^2\delta + \lambda_7\sin 2\beta\cos\delta, \\ m_{13}^2 &= -\sin\delta(2\lambda_5\sin\beta\cos\delta + \lambda_6\cos\beta), \\ m_{23}^2 &= -\sin\delta(2\lambda_5\cos\beta\cos\delta + \lambda_7\sin\beta), \\ m_{33}^2 &= 2\lambda_5\sin^2\delta, \end{aligned} \tag{16}$$

where we have used (7)–(9) to exclude $m_{1–3}$ from the equations. The value of the dipole moment of the neutron shows that we should expect δ to be relatively small [2]: $\delta < 10^{-3}\cot\beta$ and $\cot\beta \le 50$. In this case we can estimate the axion mass by means of perturbation theory with respect to $\sin\delta$. The first order correction vanishes and in the second order we get

$$m_{A_0}^2 \le 2v^2\lambda_5\sin^2\delta . \tag{17}$$

This limit cannot be removed by varying $\lambda_{6,7}$ and β. In particular, N. Maekawa's [2] suggestion concerning the possibility of existence of a heavy axion in the case of large $\tan\beta$ or $\cot\beta$ does not work, at least as long as perturbative approach is valid. In the case of MSSM, i.e. when the Higgs coupling constants are determined by SU(2) and U(1) gauge couplings g and g', the situation gets worse since $\lambda_{5–7}$ are of the order of 1-loop corrections [3]. We can therefore consider perturbation expansion with respect to them, and to the first order

$$m_{A_0}^2 \approx 2v^2\lambda_5\sin^2\delta \tag{18}$$

regardless of the value of $\sin\delta$. The numerical diagonalization of M_{ij}^2 also shows that the axion mass never exceeds the limit (18) for any β and δ, while the other bosons stay significantly heavier. This estimation is inconsistent with the recent experimental limit [2],[6] ($m_{A_0} > 20$ GeV) and we conclude that within the frames of MSSM spontaneous CP-violation is unrealistic.

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